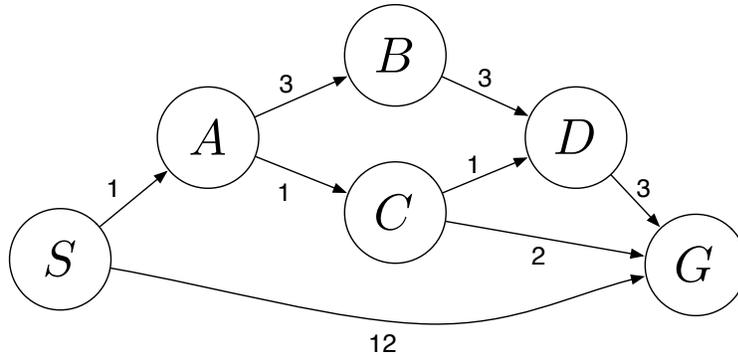


Q1. Search



Answer the following questions about the search problem shown above. Assume that ties are broken alphabetically. (For example, a partial plan  $S \rightarrow X \rightarrow A$  would be expanded before  $S \rightarrow X \rightarrow B$ ; similarly,  $S \rightarrow A \rightarrow Z$  would be expanded before  $S \rightarrow B \rightarrow A$ .) For the questions that ask for a path, please give your answers in the form ‘ $S - A - D - G$ .’

(a) What path would breadth-first graph search return for this search problem?

$S - G$

(b) What path would uniform cost graph search return for this search problem?

$S - A - C - G$

(c) What path would depth-first graph search return for this search problem?

$S - A - B - D - G$

(d) What path would A\* graph search, using a consistent heuristic, return for this search problem?

$S - A - C - G$

(e) Consider the heuristics for this problem shown in the table below.

State	$h_1$	$h_2$
$S$	5	4
$A$	3	2
$B$	6	6
$C$	2	1
$D$	3	3
$G$	0	0

- (i) Is  $h_1$  admissible? **Yes**  **No**
- (ii) Is  $h_1$  consistent? **Yes**  **No**
- (iii) Is  $h_2$  admissible?  **Yes** **No**
- (iv) Is  $h_2$  consistent? **Yes**  **No**

## Q2. Search problems

It is training day for Pacbabies, also known as Hungry Running Maze Games day. Each of  $k$  Pacbabies starts in its own assigned start location  $s_i$  in a large maze of size  $M \times N$  and must return to its own Pacdad who is waiting patiently but proudly at  $g_i$ ; along the way, the Pacbabies must, between them, eat all the dots in the maze.

At each step, all  $k$  Pacbabies move one unit to any open adjacent square. The only legal actions are Up, Down, Left, or Right. It is illegal for a Pacbaby to wait in a square, attempt to move into a wall, or attempt to occupy the same square as another Pacbaby. To set a record, the Pacbabies must find an optimal collective solution.

- (a) Define a minimal state space representation for this problem.

The state space is defined by the current locations of  $k$  Pacbabies and, for each square, a Boolean variable indicating the presence of food.

- (b) How large is the state space?

$$(MN)^k \cdot 2^{MN}$$

- (c) What is the maximum branching factor for this problem?

(A)  $4^k$    (B)  $8^k$    (C)  $4^k 2^{MN}$    (D)  $4^k 2^4$

Each of  $k$  Pacbabies has a choice of 4 actions.

- (d) Let  $MH(p, q)$  be the Manhattan distance between positions  $p$  and  $q$  and  $F$  be the set of all positions of remaining food pellets

and  $p_i$  be the current position of Pacbaby  $i$ .  
Which of the following are admissible heuristics?

$$h_A: \frac{\sum_{i=1}^k MH(p_i, g_i)}{k}$$

$$h_B: \max_{1 \leq i \leq k} MH(p_i, g_i)$$

$$h_C: \max_{1 \leq i \leq k} [\max_{f \in F} MH(p_i, f)]$$

$$h_D: \max_{1 \leq i \leq k} [\min_{f \in F} MH(p_i, f)]$$

$$h_E: \min_{1 \leq i \leq k} [\min_{f \in F} MH(p_i, f)]$$

$$h_F: \min_{f \in F} [\max_{1 \leq i \leq k} MH(p_i, f)]$$

$h_A$  is admissible because the total Pacbaby–Pacdad distance can be reduced by at most  $k$  at each time step.

$h_B$  is admissible because it will take at least this many steps for the furthest Pacbaby to reach its Pacdad.

$h_C$  is inadmissible because it looks at the distance from each Pacbaby to its most distant food square; but of course the optimal solution might have another Pacbaby going to that square; same problem for  $h_D$ .

$h_E$  is admissible because some Pacbaby will have to travel at least this far to eat one piece of food (but it's not very accurate).

$h_F$  is inadmissible because it connects each food square to the most distant Pacbaby, which may not be the one who eats it.

A different heuristic,  $h_G = \max_{f \in F} [\min_{1 \leq i \leq k} MH(p_i, f)]$ , would be admissible: it connects each food square to its closest Pacbaby and then considers the most difficult square for any Pacbaby to reach.

- (e) Give one pair of heuristics  $h_i, h_j$  from part (d) such that their *maximum* —  $h(n) = \max(h_i(n), h_j(n))$  — is an admissible heuristic.

Any pair from  $h_A, h_B$ , and  $h_E$ : the max of two admissible heuristics is admissible.

- (f) Is there a pair of heuristics  $h_i, h_j$  from part (d) such that their *convex combination* —  $h(n) = \alpha h_i(n) + (1 - \alpha) h_j(n)$  — is an admissible heuristic for any value of  $\alpha$  between 0 and 1? Briefly explain your answer.

Any pair from  $h_A, h_B$ , and  $h_E$ : the convex combination of two admissible heuristics is dominated by the max, which is admissible.

Now suppose that some of the squares are flooded with water. In the flooded squares, it takes two timesteps to travel through the square, rather than one. However, the Pacbabies don't know which squares are flooded and which aren't, until they enter them. After a Pacbaby enters a flooded square, its howls of despair instantly inform all the other Pacbabies of this fact.

- (g) Define a minimal space of belief states for this problem.

The physical states about which the agent is uncertain are configurations of  $MN$  wetness bits, of which there are  $2^{MN}$ . In general, the space of belief states would be all possible subsets of the physical states, i.e.,  $2^{2^{MN}}$  subsets of the  $2^{MN}$  configurations. However, percepts in this world give either no information about a location or perfect information, so the reachable belief states are those  $3^{MN}$  belief states in which each square is wet, dry, or unknown. Either answer is OK.

- (h) How many possible environmental configurations are there in the initial belief state, before the Pacbabies receive any wetness percepts?

$2^{MN}$

- (i) Given the current belief state, how many different belief states can be reached in a single step?

(A)  $4^k$  (B)  $8^k$  (C)  $4^k 2^{MN}$  (D)  $4^k 2^4$

After each of  $4^k$  joint movements of Pacbabies, there are  $2^k$  possible joint percepts, each leading to a distinct belief state.