CS 188
Summer 2019

Section Handout 6 Solutions

Q1. Pacman with Feature-Based Q-Learning

We would like to use a Q-learning agent for Pacman, but the size of the state space for a large grid is too massive to hold in memory. To solve this, we will switch to feature-based representation of Pacman's state.

(a) We will have two features, F_g and F_p, defined as follows:

$$F_g(s,a) = A(s) + B(s,a) + C(s,a)$$
$$F_p(s,a) = D(s) + 2E(s,a)$$

where

$A(s)$ = number of ghosts within 1 step of state s
$B(s,a)$ = number of ghosts Pacman touches after taking action a from state s
$C(s,a)$ = number of ghosts within 1 step of the state Pacman ends up in after taking action a
$D(s)$ = number of food pellets within 1 step of state s
$E(s,a)$ = number of food pellets eaten after taking action a from state s

For this pacman board, the ghosts will always be stationary, and the action space is $\{left, right, up, down, stay\}$.

calculate the features for the actions $\in \{left, right, up, stay\}$

$$F_p(s, up) = 1 + 2(1) = 3$$
$$F_p(s, left) = 1 + 2(0) = 1$$
$$F_p(s, right) = 1 + 2(0) = 1$$
$$F_p(s, stay) = 1 + 2(0) = 1$$
$$F_g(s, up) = 2 + 0 + 0 = 2$$
$$F_g(s, left) = 2 + 1 + 1 = 4$$
$$F_g(s, right) = 2 + 1 + 1 = 4$$
$$F_g(s, stay) = 2 + 0 + 2 = 4$$

(b) After a few episodes of Q-learning, the weights are $w_g = -10$ and $w_p = 100$. Calculate the Q value for each action $\in \{left, right, up, stay\}$ from the current state shown in the figure.

$$
\begin{aligned}
Q(s, up) &= w_p F_p(s, up) + w_g F_g(s, up) = 100(3) + (-10)(2) = 280 \\
Q(s, left) &= w_p F_p(s, left) + w_g F_g(s, left) = 100(1) + (-10)(4) = 60 \\
Q(s, right) &= w_p F_p(s, right) + w_g F_g(s, right) = 100(1) + (-10)(4) = 60 \\
Q(s, stay) &= w_p F_p(s, stay) + w_g F_g(s, stay) = 100(1) + (-10)(4) = 60
\end{aligned}
$$

(c) We observe a transition that starts from the state above, s, takes action up, ends in state s' (the state with the food pellet above) and receives a reward $R(s, a, s') = 250$. The available actions from state s' are $down$ and $stay$. Assuming a discount of $\gamma = 0.5$, calculate the new estimate of the Q value for s based on this episode.

$$
\begin{aligned}
Q_{new}(s, a) &= R(s, a, s') + \gamma * \max_{a'} Q(s', a') \\
&= 250 + 0.5 * \max\{Q(s', down), Q(s', stay)\} \\
&= 250 + 0.5 * 0 \\
&= 250
\end{aligned}
$$

where

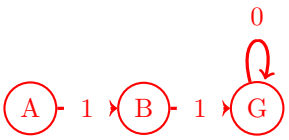
$$
\begin{aligned}
Q(s', down) &= w_p F_p(s, down) + w_g F_g(s, down) = 100(0) + (-10)(2) = -20 \\
Q(s', stay) &= w_p F_p(s, stay) + w_g F_g(s, stay) = 100(0) + (-10)(0) = 0
\end{aligned}
$$

(d) With this new estimate and a learning rate (α) of 0.5, update the weights for each feature.

$$w_p = w_p + \alpha * (Q_{new}(s,a) - Q(s,a)) * F_p(s,a) = 100 + 0.5 * (250 - 280) * 3 = 55$$
$$w_g = w_g + \alpha * (Q_{new}(s,a) - Q(s,a)) * F_g(s,a) = -10 + 0.5 * (250 - 280) * 2 = -40$$

2 Feature-Based Q-Learning

1. When using features to represent the Q-function is it guaranteed that the feature-based Q-learning finds the same optimal $Q*$ as would be found when using a tabular representation for the Q-function?
   No, if the optimal Q-function Q^* cannot be represented as a weighted combination of features, then the feature-based representation would not have the expressive power to find it. For example, consider the following MDP with deterministic transitions:



   With discount $\gamma = 1$, the optimal Q values are $Q(A, right) = 2, Q(B, right) = 1, Q(G, stay) = 0$.

   Suppose we have just one feature f, which depends only on states, with value $f(A) = 1$, $f(B) = 2$, and $f(G) = 0$. There's no linear function that can map the feature values for the states to the optimal Q values above, so it's not possible for feature-based Q-learning to find the optimal values.

Q3. MDPs and RL

Recall that in approximate Q-learning, the Q-value is a weighted sum of features: $Q(s,a) = \sum_i w_i f_i(s,a)$. To derive a weight update equation, we first defined the loss function $L_2 = \frac{1}{2}(y - \sum_k w_k f_k(x))^2$ and found $dL_2/dw_m = -(y - \sum_k w_k f_k(x)) f_m(x)$. Our label y in this set up is $r + \gamma \max_a Q(s', a')$. Putting this all together, we derived the gradient descent update rule for w_m as $w_m \leftarrow w_m + \alpha \left(r + \gamma \max_a Q(s', a') - Q(s,a)\right) f_m(s,a)$.

In the following question, you will derive the gradient descent update rule for w_m using a different loss function:

$$L_1 = \left| y - \sum_k w_k f_k(x) \right|$$

(a) Find dL_1/dw_m. Ignore the non-differentiable point.

Note that the derivative of $|x|$ is -1 if $x < 0$ and 1 if $x > 0$. So for L_1, we have:

$$\frac{\partial L_1}{\partial w_m} = \begin{cases} -f_m(x) & \text{if } y - \sum_k w_k f_k(x) > 0 \\ f_m(x) & \text{if } y - \sum_k w_k f_k(x) < 0 \end{cases}$$

(b) Write the gradient descent update rule for w_m, using the L_1 loss function.

$$\begin{aligned} w_m &\leftarrow w_m - \alpha \frac{\partial L_1}{\partial w_m} \\ &\leftarrow \begin{cases} w_m + \alpha f_m(x) & \text{if } y - \sum_k w_k f_k(x) > 0 \\ w_m - \alpha f_m(x) & \text{if } y - \sum_k w_k f_k(x) < 0 \end{cases} \end{aligned}$$

4 Probability

Use the probability table to calculate the following values:

X_1	X_2	X_3	$P(X_1, X_2, X_3)$
0	0	0	0.05
1	0	0	0.1
0	1	0	0.4
1	1	0	0.1
0	0	1	0.1
1	0	1	0.05
0	1	1	0.2
1	1	1	0.0

1. $P(X_1 = 1, X_2 = 0) = 0.15$
2. $P(X_3 = 0) = 0.65$
3. $P(X_2 = 1 | X_3 = 1) = 0.2/0.35$
4. $P(X_1 = 0 | X_2 = 1, X_3 = 1) = 1$
5. $P(X_1 = 0, X_2 = 1 | X_3 = 1) = 0.2/0.35$