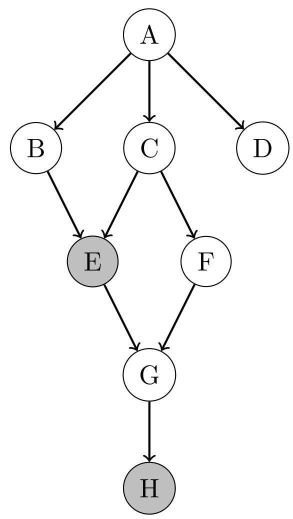
CS 188
Summer 2019

Section Handout 9

Q1. Bayes' Nets: Inference

Assume we are given the following Bayes' net, and would like to perform inference to obtain $P(B, D \mid E = e, H = h)$.



(a) What is the number of rows in the largest factor generated by *inference by enumeration*, for this query $P(B, D \mid E = e, H = h)$? Assume all the variables are binary.

○ 2^2 ○ 2^3 ○ 2^6 ○ 2^8

○ None of the above.

(b) Mark all of the following variable elimination orderings that are optimal for calculating the answer for the query $P(B, D \mid E = e, H = h)$. Optimality is measured by the sum of the sizes of the factors that are generated. Assume all the variables are binary.

☐ C, A, F, G ☐ F, G, C, A ☐ A, C, F, G ☐ G, F, C, A

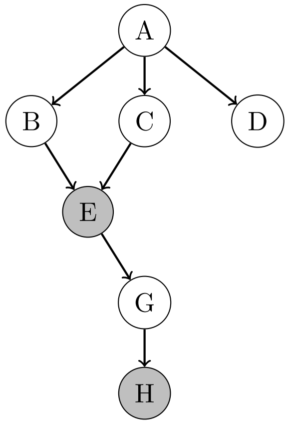
☐ None of the above.

(c) Suppose we decide to perform variable elimination to calculate the query $P(B, D \mid E = e, H = h)$, and choose to eliminate F first.

(i) When F is eliminated, what intermediate factor is generated and how is it calculated? Make sure it is clear which variable(s) come before the conditioning bar and which variable(s) come after.

$$f_1(\_\_\_\_\_\_ \mid \_\_\_\_\_\_) = \sum_f \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

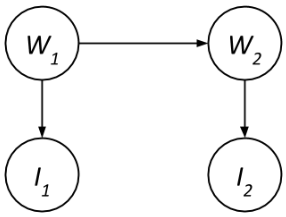
(ii) Now consider the set of distributions that can be represented by the remaining factors *after F is eliminated*. Draw the minimal number of directed edges on the following Bayes' Net structure, so that it can represent any distribution in this set. If no additional directed edges are needed, please fill in that option below.



○ No additional directed edges needed

2 Sampling and Dynamic Bayes Nets

We would like to analyze people's ice cream eating habits on sunny and rainy days Suppose we consider the weather, along with a person's ice-cream eating, over the span of two days. We'll have four random variables: W_1 and W_2 stand for the weather on days 1 and 2, which can either be rainy `R` or sunny `S`, and the variables I_1 and I_2 represent whether or not the person ate ice cream on days 1 and 2, and take values `T` (for truly eating ice cream) or `F`. We can model this as the following Bayes Net with these probabilities.



W_1	$P(W_1)$
S	0.6
R	0.4

W_1	W_2	$P(W_2 \mid W_1)$
S	S	0.7
S	R	0.3
R	S	0.5
R	R	0.5

W	I	$P(I \mid W)$
S	T	0.9
S	F	0.1
R	T	0.2
R	F	0.8

Suppose we produce the following samples of (W_1, I_1, W_2, I_2) from the ice-cream model:

`R, F, R, F` `R, F, R, F` `S, F, S, T` `S, T, S, T` `S, T, R, F`
`R, F, R, T` `S, T, S, T` `S, T, S, T` `S, T, R, F` `R, F, S, T`

1. What is $\widehat{P}(W_2 = \texttt{R})$, the probability that sampling assigns to the event $W_2 = \texttt{R}$?

2. Cross off samples above which are rejected by rejection sampling if we're computing $P(W_2 \mid I_1 = \texttt{T}, I_2 = \texttt{F})$.

   Rejection sampling seems to be wasting a lot of effort, so we decide to switch to likelihood weighting. Assume we generate the following six samples given the evidence $I_1 = \texttt{T}$ and $I_2 = \texttt{F}$:

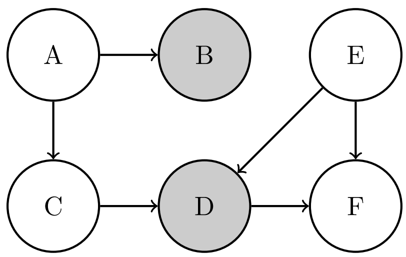
$$(W_1, I_1, W_2, I_2) = \Big\{(\texttt{S},\texttt{T},\texttt{R},\texttt{F}), (\texttt{R},\texttt{T},\texttt{R},\texttt{F}), (\texttt{S},\texttt{T},\texttt{R},\texttt{F}), (\texttt{S},\texttt{T},\texttt{S},\texttt{F}), (\texttt{S},\texttt{T},\texttt{S},\texttt{F}), (\texttt{R},\texttt{T},\texttt{S},\texttt{F})\Big\}$$

3. What is the weight of the first sample (`S`, `T`, `R`, `F`) above?

4. Use likelihood weighting to estimate $P(W_2 \mid I_1 = \texttt{T}, I_2 = \texttt{F})$.

Q3. Bayes Nets and Sampling

You are given a bayes net with the following probability tables:



E	D	F	$P(F\|E,D)$
0	0	0	0.6
0	0	1	0.4
0	1	0	0.7
0	1	1	0.3
1	0	0	0.2
1	0	1	0.8
1	1	0	0.7
1	1	1	0.3

A	$P(A)$
0	0.75
1	0.25

A	B	$P(B\|A)$
0	0	0.1
0	1	0.9
1	0	0.5
1	1	0.5

A	C	$P(C\|A)$
0	0	0.3
0	1	0.7
1	0	0.7
1	1	0.3

E	$P(E)$
0	0.1
1	0.9

E	C	D	$P(D\|E,C)$
0	0	0	0.5
0	0	1	0.5
0	1	0	0.2
0	1	1	0.8
1	0	0	0.5
1	0	1	0.5
1	1	0	0.2
1	1	1	0.8

You want to know $P(C=0|B=1,D=0)$ and decide to use sampling to approximate it.

(a) With prior sampling, what would be the likelihood of obtaining the sample [A=1, B=0, C=0, D=0, E=1, F=0]?

- ◯ 0.25*0.1*0.3*0.9*0.8*0.7
- ◯ 0.75*0.1*0.3*0.9*0.5*0.8
- ◯ 0.25*0.9*0.7*0.1*0.5*0.6
- ◯ 0.25*0.5*0.7*0.5*0.9*0.2
- ◯ 0.25*0.5*0.3*0.2*0.9*0.2
- ◯ 0.75*0.1*0.3*0.9*0.5*0.2 + 0.25*0.5*0.7*0.5*0.9*0.2
- ◯ Other ____________________

(b) Assume you obtained the sample [A = 1, B=1, C=0, D=0, E=1, F=1] through likelihood weighting. What is its weight?

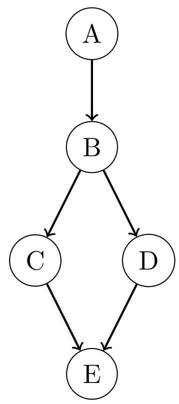
- ◯ 0.25*0.5*0.7*0.5*0.9*0.8
- ◯ 0.25*0.7*0.9*0.8 + 0.75*0.3*0.9*0.8
- ◯ 0.25*0.5*0.7*0.5*0.8
- ◯ 0
- ◯ 0.5*0.5
- ◯ 0.9*0.5 + 0.1*0.5
- ◯ Other ____________________

(c) You decide to use Gibb's sampling instead. Starting with the initialization [A = 1, B=1, C=0, D=0, E=0, F=0], suppose you resample F first, what is the probability that the next sample drawn is [A = 1, B=1, C=0, D=0, E=0, F=1]?

- ◯ 0.4
- ◯ 0.6*0.1*0.5
- ◯ 0.25*0.5*0.7*0.5*0.1*0.3
- ◯ 0.6
- ◯ 0
- ◯ 0.9*0.5 + 0.1*0.5
- ◯ Other ____________________

Q4. Bayes' Nets: Sampling

Assume we are given the following Bayes' net, with the associated conditional probability tables (CPTs).



A	**P(A)**
$+a$	0.5
$-a$	0.5

A	**B**	**P(B | A)**
$+a$	$+b$	0.2
$+a$	$-b$	0.8
$-a$	$+b$	0.5
$-a$	$-b$	0.5

B	**C**	**P(C | B)**
$+b$	$+c$	0.4
$+b$	$-c$	0.6
$-b$	$+c$	0.8
$-b$	$-c$	0.2

B	**D**	**P(D | B)**
$+b$	$+d$	0.2
$+b$	$-d$	0.8
$-b$	$+d$	0.2
$-b$	$-d$	0.8

C	**D**	**E**	**P(E | C, D)**
$+c$	$+d$	$+e$	0.6
$+c$	$+d$	$-e$	0.4
$+c$	$-d$	$+e$	0.2
$+c$	$-d$	$-e$	0.8
$-c$	$+d$	$+e$	0.4
$-c$	$+d$	$-e$	0.6
$-c$	$-d$	$+e$	0.8
$-c$	$-d$	$-e$	0.2

You are given a set of the following samples, but are not told whether they were collected with rejection sampling or likelihood weighting.

$-a$	$-b$	$+c$	$+d$	$+e$
$-a$	$+b$	$+c$	$-d$	$+e$
$-a$	$-b$	$-c$	$-d$	$+e$
$-a$	$-b$	$+c$	$-d$	$+e$
$-a$	$+b$	$+c$	$+d$	$+e$

Throughout this problem, you may answer as either numeric expressions (e.g. 0.1∗0.5) or numeric values (e.g. 0.05).

(a) Assuming these samples were generated from *rejection sampling*, what is the sample based estimate of $P(+b \mid -a, +e)$?

Answer: __________________________

(b) Assuming these samples were generated from *likelihood weighting*, what is the sample-based estimate of $P(+b \mid -a, +e)$?

Answer: __________________________

(c) Again, assume these samples were generated from *likelihood weighting*. However, you are not sure about the original CPT for $P(E \mid C, D)$ given above being the CPT associated with the Bayes' Net: With 50% chance, the CPT associated with the Bayes' Net is the original one. With the other 50% chance, the CPT is actually the CPT below.

C	**D**	**E**	**P(E | C, D)**
$+c$	$+d$	$+e$	0.8
$+c$	$+d$	$-e$	0.2
$+c$	$-d$	$+e$	0.4
$+c$	$-d$	$-e$	0.6
$-c$	$+d$	$+e$	0.2
$-c$	$+d$	$-e$	0.8
$-c$	$-d$	$+e$	0.6
$-c$	$-d$	$-e$	0.4

Samples from previous page copied below for convenience:

$-a$	$-b$	$+c$	$+d$	$+e$
$-a$	$+b$	$+c$	$-d$	$+e$
$-a$	$-b$	$-c$	$-d$	$+e$
$-a$	$-b$	$+c$	$-d$	$+e$
$-a$	$+b$	$+c$	$+d$	$+e$

Given this uncertainty, what is the sample-based estimate of $P(+b \mid -a, +e)$?

Answer: ________________________________

(d) Now assume you can only sample a *small, limited number of samples*, and you want to estimate $P(+b, +d \mid -a)$ and $P(+b, +d \mid +e)$. You are allowed to estimate the answer to one query with likelihood weighting, and the other answer with rejection sampling. In order to obtain the best estimates for both queries, *which query should you estimate with likelihood weighting*? (The other query will have to be estimated with rejection sampling.)

- ○ $P(+b, +d \mid -a)$
- ○ $P(+b, +d \mid +e)$
- ○ Either – both choices allow you to obtain the best estimates for both queries.

(e) Suppose you choose to use Gibbs sampling to estimate $P(B, E \mid +c, -d)$. Assume the CPTs are the same as the ones for parts (a) and (b). Currently your assignments are the following:

$$-a \quad -b \quad +c \quad -d \quad +e$$

(i) Suppose the next step is to resample E.
What is the probability that the new assignment to E will be +e?

Answer: ________________________________

(ii) Instead, suppose the next step is to resample A.
What is the probability that the new assignment to A will be $+a$?

Answer: ________________________________

(iii) Instead, suppose the next step is to resample B.
What is the probability that the new assignment to B will be +b?

Answer: ________________________________